

# Advertising on the Web

Hung Le

University of Victoria

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# Online Advertising

The Web gives advertisers a huge opportunity to display **relevant ads** to customers. The biggest tech companies rely on advertising as the main source of income<sup>1</sup>:

- Google: 116.3 billion US dollars in ad revenues in 2018.
- Facebook: 55 billion U.S. dollars in ad revenues in 2018.
- Ebay: 7.4 billion US dollars in marketplaces net revenue in 2018.

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<sup>1</sup>Info from <https://www.statista.com>

# Advertising Opportunities - Search Ads

Ads are placed as a part of the result for a search query. Advertisers **bid** for ad placement for each search query.

The screenshot shows a search engine interface with the query "football tickets" in the search bar. Below the search bar are navigation tabs for "All", "Images", "News", "Shopping", "Videos", "More", "Settings", and "Tools". The search results indicate approximately 1,600,000,000 results found in 0.51 seconds. The first result is an advertisement for VividSeats.com, titled "Football Tickets 2019 | Order Online - 100% Guarantee | VividSeats.com". It includes a link to www.vividseats.com/ and a description: "Great Selection of Tickets for Any Football Game Available Online! - Order Today. Email Delivery. Instant Download. Unmatched Service. Types: Instant Download Tickets, e-Mail Delivery Tickets." Below this are two sub-promotions: "100% Buyer Guarantee" (Every ticket is 100% guaranteed: Valid, Authentic & On-Time.) and "Experience Vivid Seats" (Sports, concerts, theatre? Find your tickets at Vivid Seats.). The second result is an advertisement for StubHub.ca, titled "StubHub™ Buy & Sell Tickets | Best Ticket Selection Now | StubHub.ca". It includes a link to www.stubhub.ca/ and a description: "At StubHub tickets are never sold out. Get tickets at StubHub now." Below this are several category links: "NHL Tickets · Concert Tickets · NBA Tickets · NFL Tickets · Sports Tickets · Theater and Arts Tickets". The third result is an advertisement for FootballTicketNet.com, titled "FootballTicketNet.com | Football Ticket Net | Valid & Authentic Tickets". It includes a link to www.footballticketnet.com/ and a description: "We Offer Tickets To All The Major Football Matches. 100% Order Guarantee. Secure Booking. VIP Hospitality Tickets. Seats in Pairs Guaranteed. Best Prices Online. Types: Premier League, German Bundesliga, Spanish La Liga, Champions League, Europa League, Scottish Cup, Carabao Cup." Below this are several category links: "Premier League Tickets · German Bundesliga Tickets · France Ligue 1 Tickets · Italy Serie A Tickets". The fourth result is a non-advertising link titled "Football Tickets on StubHub!" with the URL https://www.stubhub.com/football-tickets/category/31/ and a description: "Football Tickets - Buy and sell NFL and NCAA Football tickets on StubHub!". At the bottom of the page, there are navigation icons for back, forward, and search.

football tickets

All Images News Shopping Videos More Settings Tools

About 1,600,000,000 results (0.51 seconds)

**Football Tickets 2019 | Order Online - 100% Guarantee | VividSeats.com**  
[www.vividseats.com/](http://www.vividseats.com/) ▼  
Great Selection of Tickets for Any Football Game Available Online! - Order Today. Email Delivery. Instant Download. Unmatched Service. Types: Instant Download Tickets, e-Mail Delivery Tickets.

**100% Buyer Guarantee**  
Every ticket is 100% guaranteed:  
Valid, Authentic & On-Time.

**Experience Vivid Seats**  
Sports, concerts, theatre? Find  
your tickets at Vivid Seats.

**StubHub™ Buy & Sell Tickets | Best Ticket Selection Now | StubHub.ca**  
[www.stubhub.ca/](http://www.stubhub.ca/) ▼  
At StubHub tickets are never sold out. Get tickets at StubHub now.  
NHL Tickets · Concert Tickets · NBA Tickets · NFL Tickets · Sports Tickets · Theater and Arts Tickets

**FootballTicketNet.com | Football Ticket Net | Valid & Authentic Tickets**  
[www.footballticketnet.com/](http://www.footballticketnet.com/) ▼  
We Offer Tickets To All The Major Football Matches. 100% Order Guarantee. Secure Booking. VIP Hospitality Tickets. Seats in Pairs Guaranteed. Best Prices Online. Types: Premier League, German Bundesliga, Spanish La Liga, Champions League, Europa League, Scottish Cup, Carabao Cup.  
Premier League Tickets · German Bundesliga Tickets · France Ligue 1 Tickets · Italy Serie A Tickets

**Football Tickets on StubHub!**  
<https://www.stubhub.com/football-tickets/category/31/> ▼  
Football Tickets - Buy and sell NFL and NCAA Football tickets on StubHub!

# Advertising Opportunities - Display Ads

Ads are placed in many web sites. Advertisers pay for a fixed rate *per impression*.

- An impression is one display of the ad with the download of the page by some user.

The screenshot shows a Stack Overflow page with a 'Top Questions' section. The questions listed are:

- How to force DotNetCore 2.1 Web API to output Json format? What library do I need? (asked 1 min ago MC9000 542)
- React: Javascript assignment not updating object (asked 1 min ago Nate 13)
- Guest User Invitation Not Sending Email (asked 1 min ago looner 1,261)
- LineRenderer on Mesh surface (asked 1 min ago Adnan MK 59)
- I want all projects in one build (asked 1 min ago)

Two advertisements are visible on the right side of the page:

- Microsoft Azure**: Help secure and manage your Linux or Windows VMs in the cloud. Includes a 'Try Azure free' button.
- WorkSafeBc**: Government Insurance. Includes a 'Full Stack Software Developer' job listing for Richmond, BC, Canada, with tags for python, agile, and Scrum Master.

# Advertising Opportunities - Online Store Ads

Online stores, like Amazon, have many ways to show *their own ads*. The ads are chosen to maximize the buying probability.

Customers who viewed this item also viewed



AmazonBasics USB 3.0  
Cable - A-Male to Micro-B -  
6 Feet (1.8 Meters)  
★★★★☆ 82  
CDN\$ 9.13 ✓prime



USB 3.0 Micro Cable,  
Cablecreation Short USB  
3.0 A to Micro B Cord,  
Compatible External...  
★★★★☆ 29  
CDN\$ 7.99 ✓prime



AmazonBasics USB 3.0  
Cable - A-Male to Micro-B -  
3 Feet (0.9 Meters)  
★★★★☆ 122  
CDN\$ 7.83



Samsung Galaxy Note  
3/5S USB 3.0 5-Foot Data  
Cable, Non-Retail  
Packaging  
★★★★☆ 717  
CDN\$ 6.98

# Advertising Opportunities - Advertising Sites

Some web sites like Ebay or Craig's List *allow the advertisers to post their ads*. The sites can charge fee or get commission.

- ★ Feb 21 ✓ [WEBBC.ca Websites from \\$499 wordpress web design \(everywhere\)](#) [pic](#) [map](#) 
- ★ Feb 20 [EXPERT MOBILE APP & WEB DEVELOPERS](#) [437-370-2141](#) [pic](#) [map](#) 
- ★ Feb 12 [High Speed Internet, Cable and Phone deals \(Canada\)](#) [map](#) 
- ★ Jan 31 [computer REALLY REALY SLOW ? upgrade to a SSD HARD DRIVE \(Victoria\)](#) [pic](#) [map](#) 
- ★ Jan 31 [COMPUTER TEACHING FOR SENIORS \(Victoria\)](#) [pic](#) [map](#) 
- ★ Jan 31 [PROFESSIONAL COMPUTER SERVICE plus COMPUTER TEACHING for SENIORS \(VICTORIA\)](#) [pic](#) [map](#) 

# Search Ads

In this lecture, we focus mostly on search ads.

- Advertisers bid on *search queries*.
- Ads are included as a part of search results.
- If ads are clicked on, advertisers pay what they had bid.

# Challenges of Search Ads

- Ads relevance? (or when to show ad?)
- How many ads could be shown for each query?
- How to assign ads to bidder to maximize revenue without hurting user experience?
- And many more.



# The Adwords Problem

You are given:

- A set of advertisers and a set of queries. Each query is bid by a subset of advertisers, probably by different amount.
- Click through rate for each pair of advertiser-keyword.
- A budget constraint for each advertiser.
- A limit number of ads for each query.

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Your task is *for each query, when it comes in, find a set of advertisers* such that:

- The number of advertisers is smaller than the number of allowable displayed ads.
- Each advertiser has bid on the query.
- Advertisers have enough budget to pay for the ads if they are clicked on.

to **maximize the ad revenue**.

# The Simplified Adwords Problem

You are given:

- A set of advertisers and a set of queries. Each query is bid by a subset of advertisers **by the same amount**, say 1.
- Click through rates for each all advertiser-query pairs **are the same**.
- The budget of each advertiser is enough to pay for **exactly one query**. But each advertiser may bid on many queries.
- The limit number of ads for each queries is **one**.

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Your task is or each query, when it comes in, find **one** advertiser that has bid for the keyword to maximize the ad revenue.

# The Simplified Adwords Problem - Algorithm

When a new query  $q$  comes in, we do the following:

```
GREEDYSIMPLIFIEDADWORDS( $q$ )  
  if there is an unmatched advertiser  $b$   
    sell  $q$  to  $b$ .  
    mark  $b$  as matched.
```

Recall here that each advertiser  $b$  only have enough money to buy one ad for one query.

How do we know that this algorithm is good or not?

# Online vs Offline Algorithms

- Offline algorithms are given a *complete* input at front, they need to find the (optimal) output for the given input.
- Online algorithms: an input is arrived in a sequential order of its elements. The algorithm cannot see elements in the future. They need to make decision on whatever they saw so far.

# Competitive Ratio

The competitive ratio of an online algorithm  $A$ , denoted by  $CR(A)$ , intuitively is:

$$CR(A) = \min_{\text{inputs } I} \frac{\text{PERFORMANCE}(A(I))}{\text{PERFORMANCE}(B(I))} \quad (1)$$

where  $B$  is the best off-line algorithm.

# The Offline Simplified Adwords Problem

## Observation

*If all queries and their arriving orders are known beforehand, the simplified Adwords problem is equivalent to the maximum matching problem in bipartite graphs.*

A maximum matching is a matching of maximum size.



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## Observation

*The GREEDYSIMPLIFIEDADWORDS algorithm gives us a maximal matching in the queries-advertisers bipartite graph.*

A maximal matching is a matching where we cannot add an edge to it to form a new matching.

# Competitive Ratio of the Greedy Algorithm

## Theorem

*The competitive ratio of the greedy algorithm for the Simplified Adwords Problem is  $\frac{1}{2}$ .*

Upper bound proof.

See the board drawing



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## Lower bound proof.

Let  $M$  be the matching output by the greedy algorithm. Let  $M^*$  be the matching output by the optimal offline algorithm. We observe that: for every edge  $e \in M^*$ , at least one of its endpoints is matched by  $M$ . Thus,  $|M^*|$  is at most the number of matched vertices in  $M$ , which is at most  $2M$ . Thus,  $M \geq \frac{M^*}{2}$ . □

# A more practical version of the Adwords Problem

You are given:

- A set of advertisers and a set of queries. Each query is bid by a subset of advertisers **by the same amount**, say 1.
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- Each advertiser have the same amount of budget  $B$ .
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# 1st Attempt

Whenever a query  $q$  comes in, select any advertiser that has bid for this query and still have money.

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## Observation

*The competitive ratio of this algorithm is at most  $\frac{1}{2}$ .*

See the board drawing

# The Balance Algorithm

Whenever a query  $q$  comes in, select any advertiser that has bid for this query and has **largest remaining budget**.



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Whenever a query  $q$  comes in, select any advertiser that has bid for this query and has **largest remaining budget**.

## Theorem

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We will demonstrate the proof of this theorem for two advertisers.

## Theorem

*The competitive ratio of the Balance Algorithm with **two advertisers** is at least  $3/4 = 0.75$ .*

# The Balance Algorithm for Two Advertiser

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Upper bound proof.

See the board drawing □

# The Balance Algorithm for Two Advertiser

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## Lower bound proof.

See the board calculation. □

# An almost practical version of the Adwords Problem

You are given:

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# 1st Attempt - The Balance Algorithm

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# 1st Attempt - The Balance Algorithm

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## Observation

*The competitive ratio of this algorithm is arbitrarily close to 0.*

Proof. TO-DO: figure.



# The MSVV Algorithm<sup>2</sup>

Whenever a query  $q$  comes in:

- Let  $x_i$  is the current bid of the advertiser  $A_i$ .  $x_i$  could be 0.
- Let  $f_i$  be the fractional remaining budget of  $A_i$ .

We assign  $q$  to the advertiser  $A_i$  that has  $\varphi(A_i) = x_i \times (1 - e^{-f_i})$  maximum.

## Theorem

*The competitive ratio of the MSVV Algorithm is  $1 - \frac{1}{e} = 0.64$ .*

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<sup>2</sup>A. Mehta, A. Saberi, U. Vazirani, and V. Vazirani, *Adwords and generalized on-line matching*.